ComScire QNG Model R32MU
Validation Tests of Randomness
(Representative Test Results)

ComScire QNGmeter: Continuous Random Number Tester

The ComScire QNGmeter (earlier versions are <u>R</u>NGmeter) is a continuous statistical tester that uses five powerful and fundamentally different tests on the input data. Unlike other statistical test suites it is designed to measure the quality of randomness of a continuous sequence of bits up to hundreds of Terabits in length. The QNGmeter automatically performs metatests of subsequences, which would have to be done manually by other popular test suites. Every QNG Model R32MU is tested extensively after production and finally just before shipment using the QNGmeter test suite.

The five tests are:

1) 1/0 Balance – nominal expected value is $p(0) = p(1) = 0.5$.

2) Auto Correlation – orders 1 up to 32, nominal expected value is 0.0 for all orders.

3) Entropy Test – nominal expected value is H = 1.0, an update of U. Maurer's "Universal Test" (J-S Coron, *LNCS*, Vol. 1560, pp 29-42, 1999).

4) Serial Test – (Good, I. J, The serial test for sampling numbers and other tests for randomness, *Proc. Camb. Philos. Soc*. Vol. 49, 1953).

5) OQSO – Overlapping-Quadruples-Sparse-Occupancy test, nominal expected value for the mean = 141909.47* and standard deviation (by simulation) = 294.63* (G. Marsaglia and A. Zaman, *Computers Math. Applic.*, Vol. 26, No. 9, pp 1-10, 1993). *Our empirical tests indicate the actual mean and standard deviation are closer to 141909.195 and 294.656 respectively.

The z-scores, p-values, and chi-square (metatest) p-values are presented for each test. In addition, current test run time information, such as the *Bit Count*, *Elapsed Time*, and *Throughput*, is displayed by the tester. *Bit Count* is the total number of bits tested. *Elapsed Time* is the time from the start of the current test run. *Throughput* is the input data rate in bits per second.

Each test uses blocks of data of varying lengths, depending on the specific test. The 1/0 Balance and Auto Correlation tests use a block size of 65536 bits. The Serial test has a block size of 262144 bits. The Entropy test has 4194304 bits in a block. The OQSO test uses 10485775 bits per block.

A z-score is calculated for every test for each data-block. The z-scores are converted to probabilities with the assumption they are normally distributed. The z-scores of the 1/0

Balance, Auto Correlation and Serial tests and their associated p-values displayed are cumulative for all blocks. The z-scores of the Entropy and OQSO tests are combined by summing the z-scores of all blocks and dividing by the square root of the number of blocks, respectively.

A second level of testing is applied to the p-values calculated from the z-scores for each block of data. The z-scores are expected to be normally distributed and hence their associated p-values are expected to be uniformly distributed. A chi-square test is applied to the individual p-values from each of the five tests. The chi-square tests are cumulative and their results are displayed as probabilities. If these chi-square p-values converge to 0.0 or 1.0 for any test, the assumption of randomness fails, indicating non-random patterns in the data being tested.

A third level of testing is applied to all of the individual chi-squared tests. A Kolmogorov-Smirnov (KS) test is first applied to the probabilities of chi-squared results of all orders of auto correlation being tested to reduce the auto correlation results to a single number. A meta KS test is finally calculated using the auto correlation KS result and the probabilities of the chi-squared results of the remaining tests. The meta KS(+) and KS(-) probabilities are displayed. Convergence toward 1.0 or 0.0 indicates failure.

If any cumulative z-score exceeds ±4.265 standard deviations the generator will be determined as failed. Failure is also pronounced if the probability of any chi squared test is less than 0.00001 or greater than 0.99999, or if the probability of either of the Meta KS results is less than 0.0001 or greater than 0.9999.

For the hardware validation report, the QNGmeter tests were completed on a QNG Model R32MU for approximately 396 trillion random bits (396Tb). All test results for the device are recorded in the following table.

ComScire QNGmeter 396 Trillion Bits Test
Testing QNG Device S/N QWR3X003

Run Time Information		Autocorrelation	
Bits Tested	396.0E+12	**Order**	**p ($\chi2 \leq x$)**
Time Elapsed	80:23:18:00	1	0.848
Throughput	64.0E+06	2	0.850
Meter	46.5+	3	0.305
1/0 Balance		4	0.042
p ($z \leq x$)	0.552	5	0.727
p ($\chi2 \leq x$)	0.623	6	0.789
Entropy Test		7	0.804
p ($z \leq x$)	0.546	8	0.841
p ($\chi2 \leq x$)	0.061	9	0.573
Serial Test		10	0.232
p ($z \leq x$)	0.278	11	0.110
p ($\chi2 \leq x$)	0.026	12	0.585
OQSO (Monkey Test)		13	0.227
p ($z \leq x$)	0.742	14	0.657
p ($\chi2 \leq x$)	0.600	15	0.655
AC Meta KS- Test		16	0.497
KS-	0.555	17	0.848
Meta KS Test		18	0.765
KS+	0.812	19	0.047
KS-	0.276	20	0.232
		21	0.763
		22	0.174
		23	0.516
		24	0.748
		25	0.418
		26	0.335
		27	0.253
		28	0.616
		29	0.777
		30	0.954
		31	0.198
		32	0.787